Reversing Renormalization-Group Flows with AdS/CFT

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Abstract

For scalar fields in AdS with masses slightly above the Breitenlohner-Freedman bound, appropriate non-local boundary conditions can define a unitary theory. Such boundary conditions correspond to non-local deformations of the dual CFT, and generate a non-local renormalization-group flow. Nevertheless, a bulk analysis suggests that certain such flows lead to *local* CFTs in the infrared. Since the flows are non-local, they can either increase or decrease the central charge of the CFT. In fact, given any local renormalization-group flow within a certain general class which leads from a UV theory (CFT$_1$) to an IR theory (CFT$_2$), we show that one can find such a non-local flow in which the endpoints are interchanged: the non-local theory flows from CFT$_2$ in the IR to CFT$_1$ in the UV. We work at large N, but the flows we consider involve quantum field effects in the bulk, corresponding to $1/N$ corrections in the dual theory.

1 Introduction

In the AdS/CFT correspondence, boundary conditions for bulk fields are related to the specification of the dual CFT [1, 2, 3, 4]. Changes in the boundary conditions will correspond to deformations of the dual CFT Lagrangian. Deformations by relevant operators will break the conformal symmetry, generating a renormalization group flow from the original CFT in the UV. This renormalization group flow is identified via the UV/IR relation with the radial evolution of the bulk spacetime. A prototypical example of this identification is [5], where a spacetime which is asymptotically AdS both as $r \to \infty$ and as $r \to 0$ is identified with a renormalization group flow from a

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UV fixed point described by one CFT to an IR fixed point described by a different CFT. The bulk spacetime has different effective cosmological constants in the two asymptotic limits; these are identified with the central charges of the corresponding CFTs.

Bulk scalar fields in AdS_{d+1} with mass in the range $-d^2/4 \leq m^2 < -d^2/4 + 1$ provide an interesting and more subtle example of this correspondence. As indicated by the work of Breitenlohner and Freedman [6, 7], such scalar fields admit a variety of possible boundary conditions. In particular, one may fix either the faster or slower falloff part of the scalar field at infinity.

The two resulting bulk theories correspond to two different dual CFTs, in which the field ϕ is dual to operators of dimensions Δ_- and $\Delta_+ = d - \Delta_-$ respectively, where $d/2 \geq \Delta_- > d/2 - 1$. In [8, 9], it was observed that a linear boundary condition, relating the faster falloff part to the slower, corresponds to a double-trace deformation, adding a term $f\mathcal{O}^2$ to the Lagrangian of the CFT. Starting from the Δ_- CFT, this is a relevant deformation, which will produce an renormalization-group flow which is expected to end at the Δ_+ CFT in the IR; evidence for this picture has been obtained in [10, 11, 12]. Since a double-trace operator corresponds to a multiparticle state, the double-trace deformations in the CFT have also been related to worldsheet non-locality in the bulk string theory [13, 14].

This case is more subtle because pure anti-de Sitter space, with zero scalar field, remains a solution of the theory at the classical level for arbitrary linear boundary conditions on the scalar field. The existence of a non-trivial renormalization group flow is only detected from the bulk point of view once we take into account the one-loop back reaction of the scalar field, which produces a modification of the geometry depending on the boundary conditions [10].

For the classical renormalization group flows, [15, 5] established a c-theorem, demonstrating that so long as the bulk geometry satisfied a suitable energy condition, the c-function of [16] would be monotonically decreasing along the flow from the UV to the IR (see also [17]). In the case of flows associated with general linear boundary conditions for a scalar field, no such general argument exists, but [10] showed that the c-function decreases along the flow from the Δ_- CFT to the Δ_+ CFT. Thus, it seemed that the dual supergravity description automatically enforces a kind of c-theorem, agreeing with the intuitive expectation that the effective number of massless degrees of freedom decreases along the flow.

In this paper, we will show that more general boundary conditions can produce quite different behaviour. The boundary described above corresponded to local deformations of the dual CFT but, from the bulk point of view, the allowed class of boundary conditions for a scalar field (those boundary conditions that preserve the symplectic flux) is much broader. We will discuss a particular example of this broader class which corresponds to a non-local deformation in the dual CFT, and which produces a renormalization group flow which moves in the opposite direction to the one previously studied; as a consequence, the c-function increases as we flow to the IR. The bulk perspective thus indicates that, at least at large N, there are non-local deformations of the CFT which can produce controlled, well-defined renormalization group flows, and that these flows violate our usual intuition about the behaviour of

the c-function. (Since the deformation is non-local, it is perhaps not really surprising that the c-function increases; the usual intuition that the renormalization group flow corresponds to integrating out degrees of freedom on short scales does not obviously apply in this case.) It is also interesting that although the flow is non-local, the IR fixed point is a local CFT.

In the next section, we will review the constraints on allowed boundary conditions to have a well-defined bulk theory, and observe that these allow for non-local boundary conditions. In section 3, we consider a particular simple non-local boundary condition and show that the corresponding deformation of the CFT leads to a renormalization group flow from the Δ_+ CFT to the Δ_- CFT, reversing the usual flow direction. We close with some discussion in section 4.

2 Local and Non-local boundary conditions

Because AdS_{d+1} is not globally hyperbolic, boundary conditions play an especially important role in both classical and quantum dynamics. The most obvious classical role is to ensure that the initial data on a hypersurface has a unique evolution into the future. Though one could take the viewpoint that no more is required for classical physics, it is natural to also demand that the system have a well-defined phase space formulation. Such a structure is in any case pre-requisite for either canonical or path-integral quantization of the theory. We therefore restrict to boundary conditions for which the symplectic structure is finite and the symplectic flux[1] through infinity vanishes, so that the symplectic structure is conserved.

We consider only the low-energy effective theory in the bulk, which we take to be given by a local field theory including the metric as a dynamical degree of freedom. In fact, we assume that it is sufficient for our purposes to approximate even this theory by that of a linear scalar field propagating on a fixed AdS_{d+1} background. This approximation can be justified (see e.g., [20, 21]) for scalar fields with mass sufficiently close to the Breitenlohner-Freedman bound. The assumption of linearity makes the physics particularly transparent and allows us to make definite statements about the quantum theory. In section 3 below, we will also assume that the boundary conditions are linear. The more general case can then be dealt with perturbatively.

Consider such a scalar field with mass m in the range

$$-\frac{d^2}{4} + 1 > m^2 > -\frac{d^2}{4}. \tag{2.1}$$

propagating on a fixed AdS_{d+1} background,

$$ds^2 = g_{ab}dy^a dy^b = r^2(-dt^2 + \sum_{i=1}^{d-1} dx_i^2) + \frac{dr^2}{r^2}, \tag{2.2}$$

where we have restricted attention to the Poincaré patch and fixed the AdS length scale to $\ell = 1$, so $\Lambda = -\frac{1}{2}d(d-1)$.

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[1]The symplectic flux for a scalar field is proportional to the Klein-Gordon flux. See e.g. [18, 19], for general comments on symplectic structures and their role in quantization.

Since we are interested in boundary conditions, we first describe the asymptotic behavior of the field. Suppose that our scalar is associated with a potential $V(\phi)$ with squared mass $m^2 = \frac{1}{2}V''(0)$. All solutions to the equations of motion take the asymptotic form

$$\phi \rightarrow \frac{\alpha(x)}{r^{\lambda_-}} + \frac{\beta(x)}{r^{\lambda_+}}, \tag{2.3}$$

where x are coordinates on null infinity ($\partial\mathcal{M}$, also known as the conformal boundary) and where

$$\lambda_\pm = \frac{d}{2} \pm \frac{1}{2}\sqrt{d^2 + 4m^2}. \tag{2.4}$$

Note that (2.1) implies

$$2 > \lambda_+ - \lambda_- > 0. \tag{2.5}$$

The mass range (2.1) is precisely the range for which all solutions (2.3) are normalizable with respect to the symplectic structure (see e.g. [22]), except that we have excluded the special case $m^2 = -d^2/4$. Thus, the only constraint is the requirement that the symplectic flux through infinity vanish. For two vectors $\delta_1\phi, \delta_2\phi$ tangent to the space of solutions, the usual symplectic flux through a region R of null infinity is

$$\omega_R(\delta_1\phi, \delta_2\phi) = (\lambda_+ - \lambda_-)\int_R \sqrt{\Omega}(\delta_1\alpha\delta_2\beta - \delta_1\beta\delta_2\alpha), \tag{2.6}$$

where $\sqrt{\Omega}$ is the volume element on the boundary.

If our boundary condition is to force (2.6) to vanish for all regions R, then α must be an ultra-local function of β; i.e., $\alpha(x)$ can depend only on $\beta(x)$ at a point, and cannot depend on derivatives of β:

$$\alpha(x) = J_\alpha(x, \beta) \quad \text{or} \quad \beta(x) = J_\beta(x, \alpha). \tag{2.7}$$

Note that each expression (2.7) admits a potential $W_\alpha(x, \beta), W_\beta(x, \alpha)$ such that

$$\frac{\partial W_\alpha}{\partial\beta} = (\lambda_+ - \lambda_-)J_\alpha(x, \beta), \quad \frac{\partial W_\beta}{\partial\alpha(x)} = -(\lambda_+ - \lambda_-)J_\beta(x, \alpha), \tag{2.8}$$

where the normalization factor $(\lambda_+ - \lambda_-)$ on the right-hand side was chosen for later convenience. One may further show that all such boundary conditions remain valid when the scalar field is coupled to gravity; see [21] for a general analysis and [23, 24, 25, 26, 27, 20] for direct calculations.

So far, our discussion is fairly standard. We would now like to point out that while it would not correspond to our usual notion of a local bulk theory, one *could* choose to require the integrated flux (2.6) to vanish only for a certain family of regions[2] R. For example, consider a theory which associates a phase space only to spacelike surfaces Σ which are asymptotically of the form $t = constant$, independent of the spatial coordinates $\vec{x}$ on the boundary. In such a theory, the flux is required to vanish only

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[2]A different class of generalizations may be obtained by adding extra boundary degrees of freedom, and thus extra boundary terms to the symplectic structure. The study of bulk vector fields in [28] appears to be an example of this other sort of generalization.

through regions R bounded by constant t surfaces. The boundary condition $J_\alpha(x, \beta)$ may then be much more general: while we still require ultra-locality in t, one may allow the boundary condition to depend on spatial derivatives of β, or even to be a non-local function of β on each constant t surface. To emphasize the distinction between the dependence on t and $\vec{x}$, let us write such a more general boundary condition as $J_\alpha(t, \vec{x}, \beta]$.

At each t, we may allow $J_\alpha(t, \vec{x}, \beta]$ to be any functional of $\beta(\vec{x})$ for which $\frac{\delta J_\alpha(t,\vec{x},\beta]}{\delta \beta(\vec{y})}$ is the kernel of a self-adjoint operator in the L^2 space associated with the relevant constant t hypersurface. Such self-adjointness will hold when J_α can again be expressed in terms of a (perhaps non-local) potential $\mathcal{W}_\alpha(t, \beta]$ via

$$(\lambda_+ - \lambda_-)J_\alpha(t, \vec{x}, \beta] = \frac{1}{\sqrt{h}}\frac{\delta \mathcal{W}_\alpha(t, \beta]}{\delta \beta(\vec{y})}, \tag{2.9}$$

where $\sqrt{h}$ is the volume element associated with the spatial directions. Here the relation between J_α and the potential $\mathcal{W}_\alpha$ involves functional derivatives, instead of the partial derivatives of (2.8). A useful set of examples are of the form

$$\mathcal{W}_\alpha(t, \beta] = \frac{\epsilon}{2}(\lambda_+ - \lambda_-)\int_{t=const} \sqrt{h}\beta F_\gamma(\nabla)\beta, \tag{2.10}$$

where $F_\gamma(\nabla)$ is a homogeneous function of order γ in the *spatial* covariant derivatives. In general, $F_\gamma(\nabla)\beta$ may be defined by decomposing β into normal modes. The corresponding boundary condition is just

$$\alpha = \epsilon F_\gamma(\nabla)\beta. \tag{2.11}$$

Of course, a similar construction is possible for J_β and $\mathcal{W}_\beta$. For the examples (2.10) we have

$$\mathcal{W}_\beta(t, \alpha] = \frac{1}{2\epsilon}(\lambda_+ - \lambda_-)\int_{t=const} \sqrt{h}\beta[F_\gamma(\nabla)]^{-1}\beta. \tag{2.12}$$

One notes that $[F_\gamma(\nabla)]^{-1}$ is a homogeneous function of order $-\gamma$ and, as a result, at most one of $\mathcal{W}_\alpha$, $\mathcal{W}_\beta$ can be local for $\gamma \neq 0$.

Let us now consider the effect of (2.11) on the dynamics of bulk fields. It is interesting to consider the evolution of initial data of compact support. For short times, the boundary conditions will be irrelevant and the data will evolve in a purely local (and causal) fashion. However, once the initial data propagates to the boundary of AdS space, the fields will feel the effects of the boundary condition. For $\gamma \neq 0$, one expects that for at least one of α, β, the equations of motion will express 2nd time-derivatives as non-local functionals of the initial data. Thus, at this stage, the dynamics becomes non-local with respect to conformally compactified AdS (see figure 1). When this is the case, we will say that the boundary condition is "non-local," even if it may be possible to write the relation between α and β in a local form (e.g., $\alpha = \nabla^2\beta$ for $\gamma = 2$).
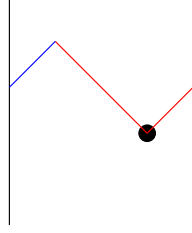
Figure 1: The effective causal structure of a theory with non-local boundary conditions. The influence of a source (black dot) in the interior propagates causally toward the boundary. However, once the signal reaches the boundary (right side), information can be instantaneously transferred elsewhere on the boundary (left side), and can then causally propagate back into the interior.

3 One-loop vacuum energy and RG flow

Let us now examine the effects of the non-locality from the boundary point of view. We first need to briefly review the relation between boundary conditions for scalar fields and the associated deformations of the dual field theory. This correspondence was conjectured in [8, 9], derived in [29], and studied further in, e.g. [30, 31, 32, 33]. The Lorentzian version of this analysis was discussed in [34], to which we refer the reader for further detail. We consider only the case of masses strictly greater than the Breitenlohner-Freedman bound, as only in this case is the UV theory exactly conformal. There are two particularly simple classes of boundary conditions, fixing α or fixing β. These two boundary conditions correspond to two different CFTs, coupled to sources of dimensions $\Delta_{\pm}$.

If we consider the boundary conditions which fix α, so that $\alpha = J_\alpha(x)$ for some fixed function J_α on $\partial\mathcal{M}$, then in the dual field theory we can consider the operator dual to the source J_α. In the (bulk) semi-classical limit its expectation value is given by

$$\langle\mathcal{O}_\alpha\rangle = \frac{1}{\sqrt{\Omega}}\frac{\delta S_{\alpha=const}}{\delta J_\alpha} = (\lambda_+ - \lambda_-)\beta. \tag{3.1}$$

Similarly, in the theory defined by fixing β, the expectation value of the corresponding operator is given by[3]

$$\langle\mathcal{O}_\beta\rangle = \frac{1}{\sqrt{\Omega}}\frac{\delta S_{\beta=const}}{\delta J_\beta} = -(\lambda_+ - \lambda_-)\alpha. \tag{3.2}$$

From (2.3), we can see that under a radial scaling $r \to \lambda r$, $\alpha \to \lambda^{\lambda_-}\alpha$, while $\beta \to \lambda^{\lambda_+}\beta$. This implies that the conformal dimensions of the corresponding operators are indeed $\Delta_+ = \Delta(\mathcal{O}_\alpha) = \lambda_+$ and $\Delta_- = \Delta(\mathcal{O}_\beta) = \lambda_-$.

Passing to more general boundary conditions will correspond to a deformation away from exact conformal invariance, generating a renormalization group flow. The deformation in the CFT action can be computed by using the bulk description to

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[3]Note that the action used here $S_{\beta=const}$ differs from $S_{\alpha=const}$ by a boundary term.

calculate the effect of the change in boundary conditions on matrix elements and then applying a Schwinger variational principle. In the semi-classical limit, the change in boundary conditions changes matrix elements in two ways: by changing the bulk solution, and by changing the appropriate bulk action; at each boundary condition, the bulk action must be chosen to give a well-defined variational principle. The result is that the deformation of the field theory is determined by the corresponding potential in (2.8). That is,

$$S^{FT} - S^{FT}_{\alpha=0} = \int dt \, \mathcal{W}_\alpha \big|_{\beta=\frac{1}{\lambda_+ - \lambda_-} \mathcal{O}_\alpha} + \mathcal{O}(1/N), \qquad (3.3)$$

and

$$S^{FT} - S^{FT}_{\beta=0} = \int dt \, \mathcal{W}_\beta \big|_{\alpha=\frac{-1}{\lambda_+ - \lambda_-} \mathcal{O}_\beta} + \mathcal{O}(1/N), \qquad (3.4)$$

where in each case, the argument of the potential is taken to be the appropriate CFT operator.

Let us consider a particularly simple example of a non-local boundary condition, corresponding to $\gamma = -2$ in (2.11). The boundary condition imposes a mode-dependent relation between the Fourier coefficients in the expansion of the functions $\alpha(x)$ and $\beta(x)$ in Fourier modes with respect to the *spatial* coordinates $\vec{x}$. For simplicity, we will consider a linear relation,

$$\alpha_k(t) = g\vec{k}^{-2}\beta_{\vec{k}}(t), \qquad (3.5)$$

for some constant g. In position space, this boundary condition would correspond to $\alpha(x) = g\nabla^{-2}\beta(x)$, where ∇^2 is the spatial Laplacian.

The boundary condition (3.5) thus corresponds to deforming the Δ_+ CFT by the relevant, non-local operator

$$\mathcal{W}_\alpha = \frac{g}{\lambda_+ - \lambda_-} \int_{t=const} \sqrt{h} \, \tilde{\mathcal{O}}\nabla^{-2}\tilde{\mathcal{O}}. \qquad (3.6)$$

The Δ_+ CFT corresponds to $\alpha = 0$ boundary conditions in the bulk[4]. Since this is a relevant deformation, the coupling g has a positive conformal dimension. From the bulk point of view, there is a natural prediction for the effect of this perturbation: in the UV, $g \to 0$, and we will recover the Δ_+ CFT $\alpha_k = 0$. In the IR, $g \to \infty$, and we expect to find the Δ_- CFT $\beta_k = 0$.

The roles of the Δ_+ and Δ_- CFTs are thus reversed compared to an ultra-local deformation. Since [10] showed that $c_- > c_+$, the central charge increases along our renormalization group flow. We can see how this happens by following though the details of the calculation of the (position-dependent) correction to the bulk vacuum energy as in [10]. The analysis of the effect of the boundary conditions on the propagator was carried out mode by mode in [10], so by substituting $f = \vec{k}^2/g = r^2\tilde{k}^2/g$ in their eq. (35), we see immediately that

$$V(x, g) - V(x, \infty) = -\frac{1}{2^{d-2}\pi^{\frac{d}{2}}\Gamma(\frac{d}{2})} \int_0^\nu d\tilde{\nu}\frac{\tilde{\nu}}{\Gamma(\tilde{\nu})\Gamma(1-\tilde{\nu})} \int_0^\infty d\tilde{k}\frac{\tilde{k}^d}{\tilde{k}+\tilde{g}}[K_\nu(\tilde{k})]^2, \quad (3.7)$$

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[4]Note that our notation differs from that of [10].

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where $\nu = \sqrt{m^2 - m_{BF}^2}$, and

$$\tilde{g} = 2^{-2\nu} \frac{\Gamma(1-\nu)}{\Gamma(1+\nu)} g \left(\frac{1}{r}\right)^{2(1-\nu)} \tag{3.8}$$

contains all of the position dependence. The integral on the RHS is negative, and is a monotonic function of $\tilde{g}$. Hence, we can see that $V(x, g)$ is a monotonically increasing function of $\tilde{g}$, reaching its maximum value at $\tilde{g} = \infty$. That is, the correction to the cosmological constant increases towards the interior of the spacetime, corresponding to an increasing central charge in the IR.

4 Discussion

Our comments above considered non-local boundary potentials $\mathcal{W}_\alpha$, $\mathcal{W}_\beta$ in the context of AdS/CFT. For the particular boundary condition $\alpha = g\nabla^{-2}\beta$, where ∇^2 is the spatial Laplacian, we computed the position-dependent one-loop vacuum energy following [10] and found that the corresponding renormalization group flow runs from the Δ_+ CFT in the UV to the Δ_- CFT in the IR. As a result, the effective central charge *increases* along this flow, and the endpoints of the flow are interchanged relative to flow associated with the more familiar ultra-local boundary conditions. This is consistent because the UV description of our flow is given by a non-local perturbation of the Δ_+ CFT.

The above example is a special case of the boundary condition (2.11) with $\gamma = -2$. Let us briefly consider the more general case, but restrict ourselves to F_γ such that either F_γ or $[F_\gamma]^{-1}$ is local. This requires $\gamma \in \mathbb{Z}$. The case $\gamma = 0$ is familiar, and for the remaining cases only one of $\mathcal{W}_\alpha, \mathcal{W}_\beta$ can be local. As usual, we restrict to the mass range (2.1). For $\gamma \geq 2$, the local potential is $\mathcal{W}_\beta$ and is irrelevant. Here the effective central charge decreases along the RG flow. For $\gamma \leq -1$ the local potential is $\mathcal{W}_\alpha$ and is again irrelevant. Such cases are like the example of section 3, and the effective central charge increases along the RG flow. The remaining case $\gamma = 1$ is more interesting, however. Here the local potential is $\mathcal{W}_\beta$. For small mass $(m^2 - m_{BF}^2 < 1/4)$, this potential is irrelevant. However, it becomes marginal when $m^2 - m_{BF}^2 = 1/4$, and then relevant for larger masses. We see that when both $F_1 = v^i\nabla_i$ for some vector field v^i and $m^2 - m_{BF}^2 \geq 1/4$, the boundary dual admits a UV-complete local description. This suggests that the dynamics of bulk fields should be fully local on conformally compactified AdS. While it is not apparent to us what makes such cases special from the bulk perspective, it is clear that further study is warranted. The case $m^2 - m_{BF}^2 = 1/4$ is particularly interesting, as it arises in the usual maximal gauged supergravity theory on AdS$_4$ [35, 36], which can in fact be consistently truncated to a theory involving only gravity and this scalar [37].

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